PRICE DIFFERENCE ON SEMI-ANNUAL COUPON VS ANNUAL COUPON BONDS WITH SAME QUOTED YTM

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LET F = FACE, C = ANNUAL COUPON RATE, Y = ANNUAL YTM, & N = YEARS TO MATURITY. PRICE AT A COUPON DATE.

PRICE (SEMI-ANNUAL) − PRICE (ANNUAL)

$$= \frac{C/2 \cdot F}{Y/2}\left[1 - \frac{1}{\left(1+\frac{Y}{2}\right)^{2N}}\right] + \frac{F}{\left(1+\frac{Y}{2}\right)^{2N}} - \left\{\frac{C \cdot F}{Y}\left[1 - \frac{1}{(1+Y)^N}\right] + \frac{F}{(1+Y)^N}\right\}$$

$$= \frac{F \cdot C}{Y}\left[-\frac{1}{\left(1+\frac{Y}{2}\right)^{2N}}\right] + \frac{F}{\left(1+\frac{Y}{2}\right)^{2N}} - \left\{\frac{F \cdot C}{Y}\left[-\frac{1}{(1+Y)^N}\right] + \frac{F}{(1+Y)^N}\right\}$$

$$= F\left(\frac{C}{Y} - 1\right)\left[\frac{1}{(1+Y)^N} - \frac{1}{\left(1+\frac{Y}{2}\right)^{2N}}\right], \quad Y \neq 0. *$$

- $\frac{C}{Y} - 1$: ⊕ FOR PREMIUM BONDS, IF $Y > 0$; ⊖ FOR DISCOUNT BONDS, IF $Y > 0$; ZERO FOR PAR BONDS.
- $\left[\frac{1}{(1+Y)^N} - \frac{1}{\left(1+\frac{Y}{2}\right)^{2N}}\right]$: • STRICTLY POSITIVE IF $Y \neq 0$; • ZERO IF $Y = 0$

* IF $Y < 0$, $\frac{C}{Y} - 1 < 0$, $\left[\frac{1}{(1+Y)^N} - \frac{1}{\left(1+\frac{Y}{2}\right)^{2N}}\right] > 0$, AND PRICE DIFFERENCE < 0.

* IN CASE $Y = 0$, THESE FORMULAE DO NOT APPLY BUT TVM MATH GIVES PRICE DIFFERENCE OF ZERO.